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Spacecraft Momentum Dumping Using Fewer than Three External Control Torques

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Momentum wheels are used in spacecraft attitude control systems to stabilize the attitude of the spacecraft. Disturbance torques on the spacecraft increase its angular momentum and speed up the momentum wheels. The angular momentum stored in the momentum wheels needs to be removed by implementing external torques to prevent saturation. Control algorithms are developed in this paper that remove the accumulated angular momentum, despite the loss of external control torque about one or even two of the principal axes of the spacecraft due to failures. The procedure consists of three stages. First, the angular speed of the momentum wheels corresponding to the unactuated axes is removed by performing attitude maneuvers. The angular momentum stored in the remaining wheels is then dumped using a conventional momentum-dumping method. Finally, the desired attitude of the spacecraft is restored. Numerical simulations demonstrate the successful and smooth performance of the proposed control algorithm and the control laws involved.

Nomenclature

$[D]$	=	derivative gain matrix
$\mathbf{H}_c$	=	total angular momentum vector of the spacecraft and the wheels about the center of mass
$[I^*]$	=	total mass moment of inertia tensor of the spacecraft and the wheels
$[I^{\Delta}]$	=	effective inertia tensor
$[K]$	=	proportional gain matrix
$\mathbf{q}$	=	quaternion vector of the spacecraft
$[q]$	=	the first three components of the quaternion vector
$\mathbf{T}_c$	=	external torque vector on the spacecraft about the center of mass
$\mathbf{u}$	=	motor torques vector of the momentum wheels
$\mathbf{\Omega}$	=	spinning speed vector of the momentum wheels relative to spacecraft
$\boldsymbol{\omega}$	=	angular velocity vector of the spacecraft

I. Introduction

THRUSTERS perform a key role in spacecraft momentum-dumping procedures. Any external torque, including disturbance torques, changes the overall angular momentum of the spacecraft. Momentum wheels (MWs) apply internal torques to the body of the spacecraft. When momentum wheels are activated for controlling the spacecraft, the change of angular momentum caused by the disturbance torques is transferred to the momentum wheels, altering their spin speed so that the spacecraft attitude is unchanged. Over time, the momentum wheels' angular velocity approaches the saturation limit. External torque actuators, specified in the next paragraph, are used to remove the angular momentum accumulated in the flywheels to prevent saturation.

Three sources of external torque are most commonly used for this procedure [1,2]: gravity gradient torque, magnetic torque, and the torque provided by attitude thrusters. In this paper, we consider attitude thrusters as the source of the external torque required for momentum dumping.

Specifically, we examine instances where one or two of the spacecraft's thrusters have failed and are out of order. If the control torque over one of the spacecraft's main axes is lost, the spin speed of the corresponding wheel cannot be changed, and the wheels cannot be desaturated by existing methods. The proposed control algorithm first removes the angular momentum of the momentum wheel(s) corresponding to the faulty thruster(s) through the use of gyric moment during some well-designed attitude maneuvers. In the next step, the remaining components of the spacecraft's angular momentum are dumped using the remaining thrusters. Numerical simulations demonstrate the satisfactory performance of the controller.

II. Governing Equations

The governing dynamic equations include kinetic equations relating torque to angular velocities, and kinematic equations relating angular velocities to attitude. We first write the relationship between total angular momentum of the spacecraft and external torques. It is assumed that there are three identical momentum wheels having their respective rotational axes along the three principal axes of the body of the spacecraft. It is also assumed that the principal axes of the body of the spacecraft coincide with the vehicle axes about which the torques are applied. The moment of inertia of the ith momentum wheel about its motor axis is J_i. The matrix $[J]$ is defined as

$$[J] = \begin{bmatrix} \ddots & & 0 \\ & J_i & \\ 0 & & \ddots \end{bmatrix} \tag{1}$$

Skew symmetric matrix format $[\tilde{a}]$ is used to put the cross product into matrix form [3]; $\mathbf{a} \times \mathbf{b} = [\tilde{a}]\mathbf{b}$.

$$\mathbf{H}_c = [I^*]\boldsymbol{\omega} + [J]\mathbf{\Omega} \tag{2a}$$

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$$[I^*] = \begin{bmatrix} I_1^* & 0 & 0 \\ 0 & I_2^* & 0 \\ 0 & 0 & I_3^* \end{bmatrix} \tag{2b}$$

$$I^*_{i(\text{total})} = I_{i(\text{body})} + J_{i(\text{wheel, axial})} + 2I_{t(\text{wheels, transverse})}$$

$$\dot{\mathbf{H}}_c = -\boldsymbol{\omega} \times \mathbf{H}_c + \mathbf{T}_c \tag{2c}$$

$$[I^*]\dot{\boldsymbol{\omega}} + [J]\dot{\boldsymbol{\Omega}} + [\tilde{\omega}]\mathbf{H}_c = \mathbf{T}_c \tag{3}$$

Ω_i is the spinning speed of the ith momentum wheel relative to the spacecraft and $\Omega = [\cdots \quad \Omega_i \quad \cdots]^T$. The dynamic equations relating the torques produced by the momentum wheel motors to the speed of the wheels are also considered, as follows:

$$[J]\{\dot{\boldsymbol{\Omega}} + \dot{\boldsymbol{\omega}}\} = \mathbf{u} \tag{4}$$

Combining Eqs. (3) and (4) results in our final set of kinetic equations:

$$\dot{\boldsymbol{\Omega}} = -\dot{\boldsymbol{\omega}} + [J]^{-1}\mathbf{u} \tag{5}$$

$$\dot{\boldsymbol{\omega}} = [I^{\Delta}]^{-1}\{-\mathbf{u} - [\tilde{\omega}]\mathbf{H}_c + \mathbf{T}_c\} \tag{6}$$

$$[I^{\Delta}] = \{[I^*] - J\} \tag{7}$$

The quaternion vector $\mathbf{q} = [q_1, q_2, q_3, q_4]^T$ is used to represent the attitude of the spacecraft. Defining $[q] = [q_1, q_2, q_3]$, the kinematic equation describing attitude changes is [4]

$$\frac{\mathrm{d}}{\mathrm{d}t}[q] = -\frac{1}{2}\boldsymbol{\omega} \times [q] + \frac{1}{2}q_4\boldsymbol{\omega} \tag{8a}$$

$$\dot{q}_4 = -\frac{1}{2}\boldsymbol{\omega}^T[q] \tag{8b}$$

The governing equations of the system comprise kinematic Eqs. (8a) and (8b) and kinetic Eqs. (5) and (6).

III. Momentum-Dumping Algorithms

During the momentum-dumping process, the angular velocity of the spacecraft's momentum wheels needs to be decreased. The initial conditions of the momentum-dumping process are identified as the spacecraft being in the desired orientation but with momentum wheels rotating near saturation speeds. The desired final condition is to have the spacecraft in the desired orientation and the speeds of the momentum wheels lowered to the specified values. The desired orientation is assumed to be fixed (nonrotating) in the inertial frame.

We consider attitude thrusters for applying external torques on the spacecraft. The thrusters should be implemented in pairs to apply pure torque on the spacecraft. If one of the thrusters fails, we cannot apply a "pure" torque about a certain principal axis. It is assumed that the basic external torque axes coincide with the principal axes of the spacecraft. When we cannot apply external torque using thrusters about a certain principal axis, we call that axis *unactuated*.

Equation (2c) reveals that, in addition to external control torques, gyric torque ($-\boldsymbol{\omega} \times \mathbf{H}_c$) can be used to change the body-fixed angular momentum vector. The following two subsections explain certain attitude maneuvers that result in decreasing the components of the $\mathbf{H}_c$ along the unactuated axes to zero using gyroscopic effects. Because only momentum wheels are used for these maneuvers, the angular momentum in the unactuated momentum wheels is transferred to the *actuated* wheels, resulting in an increase in their speeds. After making sure that $\mathbf{H}_c$ only has components about the actuated axes, we will use conventional momentum-dumping methods, as explained in Sec. V, to completely remove the angular momentum of the satellite.

When a momentum wheel becomes saturated, it cannot generate any torque that would increase its speed further, but it can produce a torque that reduces its speed. Also, because the total momentum is inertially fixed, the stated maneuver only redistributes the angular momentum and may increase the spinning speed of actuated wheels. The actuated wheels speeds, therefore, should be at a safe margin from saturation so that they do not saturate in this step.

A. Momentum Dumping Using Two External Control Torques

In this section, we assume that the third principal axis is unactuated. The axis z is the third principal axis in the body-fixed spacecraft coordinate system. Because only momentum wheels are used in this step, the angular momentum of the satellite does not change in the inertial coordinate system. To remove $(\mathbf{H}_c)_3$, the third component of the angular momentum vector in the body-fixed coordinate system, the spacecraft is turned such that its z axis becomes normal to the angular momentum vector. The body of spacecraft is now fixed and matrix $[J]$ is diagonal, therefore, the vanishing of $(\mathbf{H}_c)_3$ is equivalent to vanishing of Ω_3. The initial direction of the z axis is called $\mathbf{Z}_0$ and the projection of $\mathbf{Z}_0$ on the plane normal to $\mathbf{H}_c$ is named $\mathbf{Z}'_f$. The shortest path to rotate the spacecraft such that its z axis lies in the plane normal to $\mathbf{H}_c$, which remains fixed in space, is to align z with $\mathbf{Z}'_f$, as shown in Fig. 1. Note that, in contrast with $\mathbf{Z}_0$, $\mathbf{Z}'_f$ is not a unit vector, and the unit vector parallel to it is called $\mathbf{Z}_f$. $\mathbf{Z}_f$ will be the direction of the z axis after the spacecraft is rotated. The shortest rotation from $\mathbf{Z}_0$ to $\mathbf{Z}_f$ is done about $\mathbf{Z}_0 \times \mathbf{Z}_f$, as illustrated in Fig. 1. The spacecraft is rotated about the unit vector parallel to $\mathbf{Z}_0 \times \mathbf{Z}_f$, and the rotation angle α is the angle between the two vectors $\mathbf{Z}_0$ and $\mathbf{Z}_f$.

As illustrated in Fig. 1, the final direction of the z axis $\mathbf{Z}_f$ and the rotation angle α can be derived as follows [5]:

$$\mathbf{Z}'_f = \mathbf{Z}_0 - \frac{\mathbf{H}_c \cdot \mathbf{Z}_0}{|\mathbf{H}_c|}\frac{\mathbf{H}_c}{|\mathbf{H}_c|}, \qquad \mathbf{Z}_f = \frac{\mathbf{Z}'_f}{\mathbf{Z}'_f} \tag{9}$$

$$\alpha = \sin^{-1}(|\mathbf{Z}_0 \times \mathbf{Z}_f|) \tag{10}$$

The quaternion vector corresponding to the final attitude relative to $\mathbf{Z}_0$ will therefore be

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \frac{(\mathbf{Z}_0 \times \mathbf{Z}_f)}{|\mathbf{Z}_0 \times \mathbf{Z}_f|}\sin(\alpha/2) \tag{11a}$$

$$q_4 = \cos(\alpha/2) \tag{11b}$$

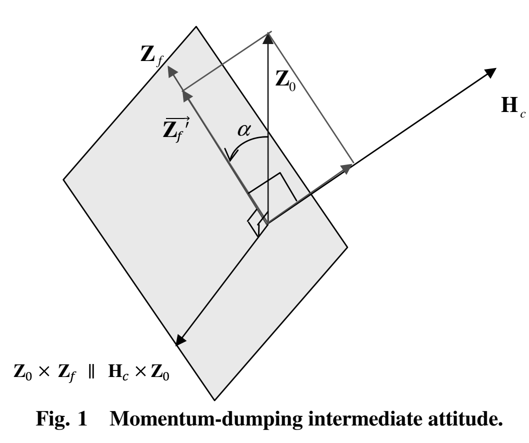


Fig. 1 Momentum-dumping intermediate attitude.

B. Momentum Dumping with One External Control Torque

The accumulated momentum in the momentum wheels can be removed using even one external control torque. This procedure may become necessary if subsequent thruster failures result in two unactuated axes. The general solution is to first rotate the spacecraft so that the actuated axis is aligned with the direction of the total angular momentum vector. In the process, this alignment removes the angular momentum of the wheels coincident with the two unactuated principal axes. In the next step, the available thrusters are used to exert a torque in the reverse direction of the momentum vector, which is now in the same direction as the actuated axis. After all the angular momentum of the spacecraft has been removed, the spacecraft is returned to its initial attitude, implementing the momentum wheels and using the same control scheme for attitude stabilization as in the previous two steps. Without loss of generality, the x axis is assumed to be the external torque-actuated axis of the spacecraft. The shortest way to bring $\hat{e}_1 = [1, 0, 0]^T$, the unit vector along the x axis, along the $\mathbf{H}_c$, the angular momentum vector, is through rotation of the spacecraft about their cross product $\hat{e}_1 \times \mathbf{H}_c$. The rotation angle α can be found from the following equations:

$$\sin(\alpha) = \frac{|\hat{e}_1 \times \mathbf{H}_c|}{|\mathbf{H}_c|} \tag{12}$$

$$\cos(\alpha) = \frac{\hat{e}_1 \cdot \mathbf{H}_c}{|\mathbf{H}_c|} \tag{13}$$

The quaternions representing the intermediate coordinate system are

$$\begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} = \frac{\hat{e}_1 \times \mathbf{H}_c}{|\hat{e}_1 \times \mathbf{H}_c|} \sin\left(\frac{\alpha}{2}\right) \tag{14}$$

$$q_4 = \cos(\alpha/2) \tag{15}$$

Now that the quaternions associated with the intermediate attitude have been derived, a control law must be implemented to bring the attitude of the spacecraft to the desired setting using the momentum wheels.

IV. Control Methods for Changing Attitude

In this section, we present a control method for performing attitude-changing maneuvers. By inspecting Eq. (6), we find that the momentum wheels torque vector $\mathbf{u}$, if negated, will act as external torque $\mathbf{T}_c$. The quaternion vector describing the attitude of the spacecraft relative to the desired orientation is denoted as $\mathbf{q}_{\text{rel}}$; $q_{4\text{rel}}$ and $[q_{\text{rel}}]$ are defined correspondingly. The control method developed was inspired by Vadali and Junkins [6], and is formulated as follows [5]:

$$-\mathbf{u} = -[K]q_{4\text{rel}}[q_{\text{rel}}] - [D]\boldsymbol{\omega} + [\tilde{\omega}]\mathbf{H}_c \tag{16}$$

The preceding control law makes $\mathbf{q}_{\text{rel}} = [0, 0, 0, 1]^T$ globally stable, which means that the attitude will eventually converge to its prescribed orientation, irrespective of initial conditions or control gains. Prescribed motor torques are independent of the sign of the quaternion vector; $\mathbf{q}_{\text{rel}}$ and $-\mathbf{q}_{\text{rel}}$ represent the same attitude. (In fact, the rotation angle of $-\mathbf{q}_{\text{rel}}$ is 2π plus that of $\mathbf{q}_{\text{rel}}$, whereas the unit vector of both quaternions are identical.) Since $q_{4\text{rel}}[q_{\text{rel}}] = -q_{4\text{rel}}[-q_{\text{rel}}]$, the control torques resulting from Eq. (16) will be the same for $\mathbf{q}_{\text{rel}}$ and $-\mathbf{q}_{\text{rel}}$.

In our numerical simulations, we have chosen $[K] = k[I^{\Delta}]$ and $[D] = d[I^{\Delta}]$, k and d being the control scalar gains. Using Eqs. (6) and (16), the angular acceleration of the body of the spacecraft can be calculated as

$$\dot{\boldsymbol{\omega}} = -kq_{4\text{rel}}[q_{\text{rel}}] - d\boldsymbol{\omega} \tag{17}$$

V. Angular Momentum Removal Using External Control Torques

In this section, we present a control law to determine the external control torque required to remove the angular momentum of the spacecraft. The main idea is to stabilize the attitude of the spacecraft using the momentum wheels, while exerting the appropriate external torques to reduce the angular momentum. When the spacecraft is stabilized using the momentum wheels, all the external torques are transferred to the momentum wheels to prevent rotation of the spacecraft. We assign this external torque to be negatively proportional to the angular momentum vector of the vehicle:

$$\mathbf{T}_c = -k_{\text{dump}}\mathbf{H}_c \tag{18}$$

In the preceding equation, k_{dump} is the positive scalar gain. Provided the spacecraft is not rotating, implementing Eq. (18) in Eq. (2) results in

$$\dot{\mathbf{H}}_c = [-k_{\text{dump}}I_{3\times3}]\mathbf{H}_c \tag{19}$$

Since $[-k_{\text{dump}}I_{3\times3}]$ is a negative definite matrix, with three eigenvalues being equal to $-k_{\text{dump}}$, the angular momentum of the spacecraft converges exponentially to zero.

The momentum removal using external torques (produced, for example, by thrusters) is performed in the second phase of momentum dumping. By then, only actuated wheels are spinning and angular momentum vector $\mathbf{H}_c$ has zero components along unactuated axes. The torque required by Eq. (18) for momentum dumping will only have components about the actuated axis. Corresponding thrusters are not affected by the failure and can provide the required torque. Also, because the spacecraft has already been turned to the fixed intermediate attitude, it does not rotate and fulfills the assumption made in the derivation of Eq. (18).

VI. Numerical Simulations and Discussion

Numerical simulations were carried out to demonstrate the performance of the control laws in action. The specifications of the spacecraft are assumed to be as follows:

$I_1 =$ the moment of inertia of the body of the spacecraft about the x axis $= 86.215\ \text{kg} \cdot \text{m}^2$.

$I_2 =$ the moment of inertia of the body of the spacecraft about the y axis $= 85.07\ \text{kg} \cdot \text{m}^2$.

$I_3 =$ the moment of inertia of the body of the spacecraft about the z axis $= 113.565\ \text{kg} \cdot \text{m}^2$.

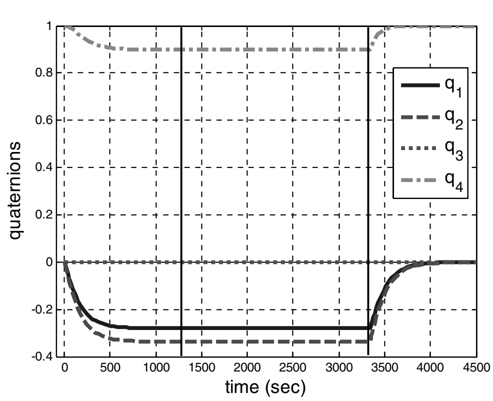
$J_a =$ axial moment of inertia of each wheel $= 0.05\ \text{kg} \cdot \text{m}^2$.

Torque limit of the thrusters $=$ torque limit of the wheel motors $= 0.2\ \text{N} \cdot \text{m}$.

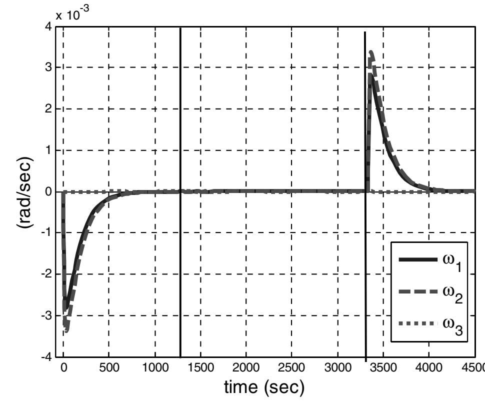
The control gains k and d, previously defined in Sec. IV, are selected as 0.0013 and 0.105, respectively, to observe the torque limit of the momentum wheels. As a result, the time constant of attitude convergence in slewing maneuvers (phases 1 and 3 of the simulations) is about 169 s. The saturation speed of the wheels is assumed to be 1000 rad/s. Throughout the numerical simulations, thruster torques were assumed to be continuous. In reality, thrusters can be either on or off, and their torques cannot vary continuously. The pulse width modulation method is used to turn the thrusters on or off periodically such that the average torque on the spacecraft is the prescribed torque [2].

A. Momentum Dumping Using Two External Control Torques

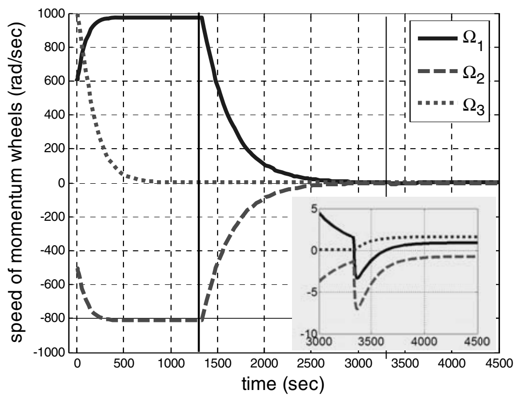
Figure 2 illustrates the simulation results for momentum dumping of the spacecraft using the available thruster torques about two axes. The initial condition is considered as the spacecraft axes being coincident with the inertial coordinate system with zero rates and the momentum wheels rotating at high spin rates. The momentum



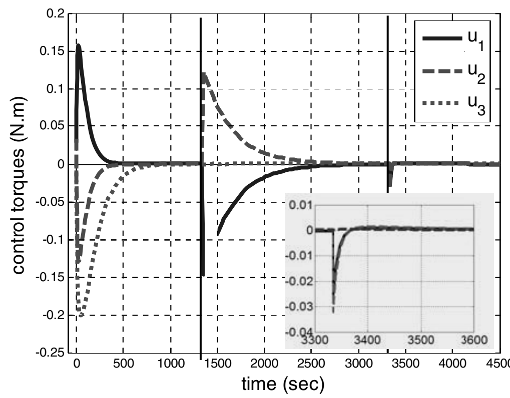
a) **Attitude of the body**



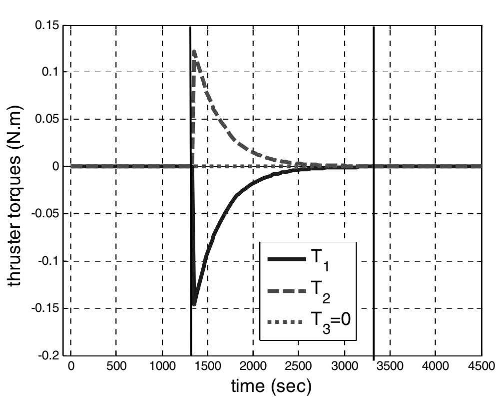
b) **Angular velocity of the body**



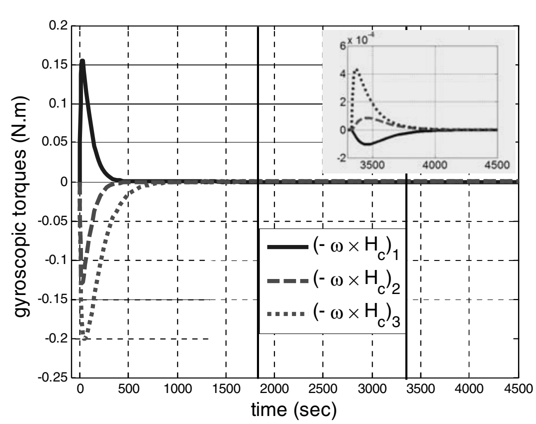
c) **Spin rate of MWs**



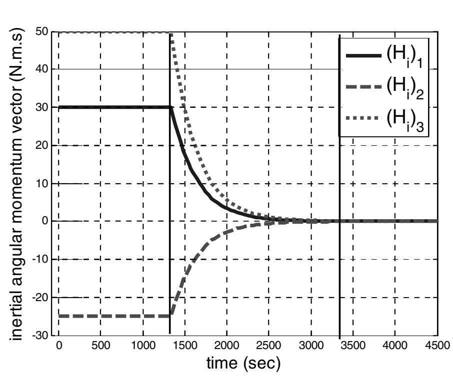
d) **Torques of the MW motors**



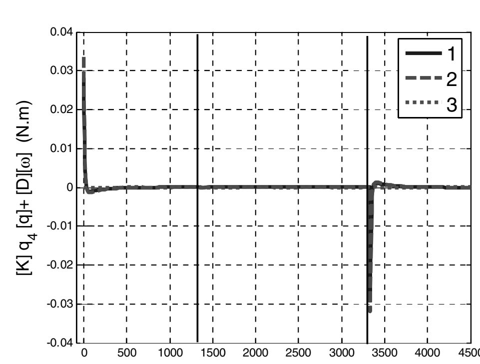
e) **Thruster control torques**



f) **Gyric torques**

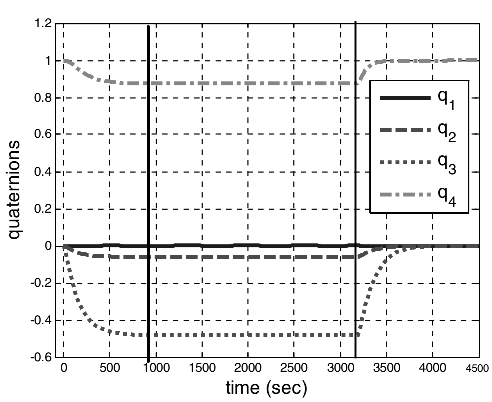


g) **Angular momentum in the inertial frame**

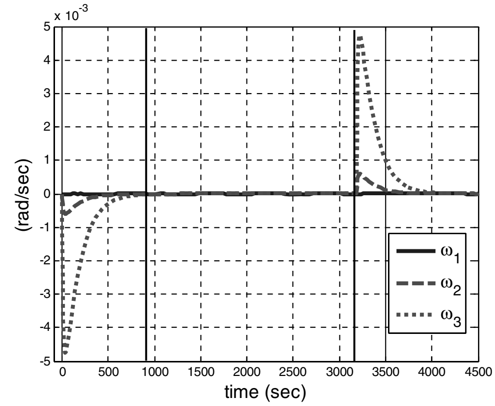


h) **MW torques, gyric torque subtracted**

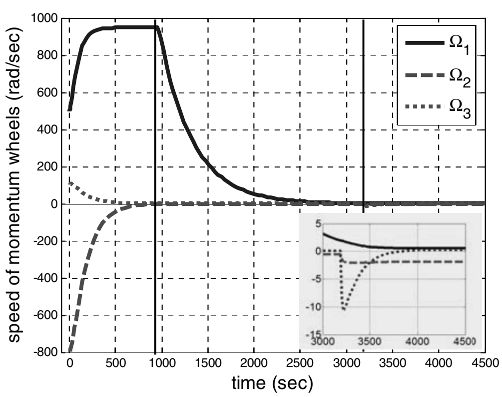
Fig. 2 Simulation results for momentum dumping of spacecraft using available thruster torques about two axes.



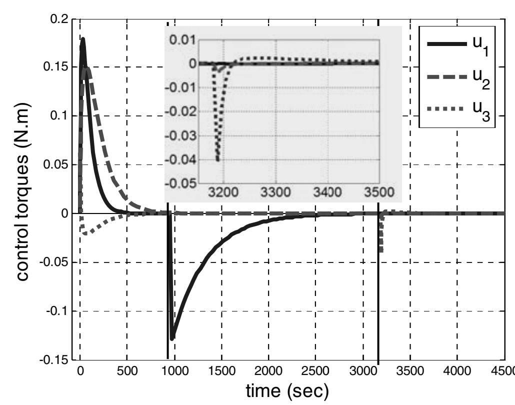
a) Attitude of the body



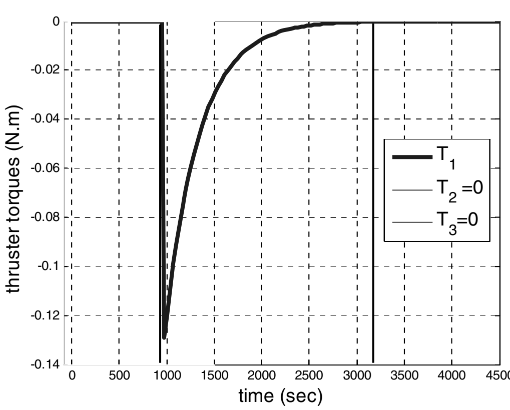
b) Angular velocity of the body



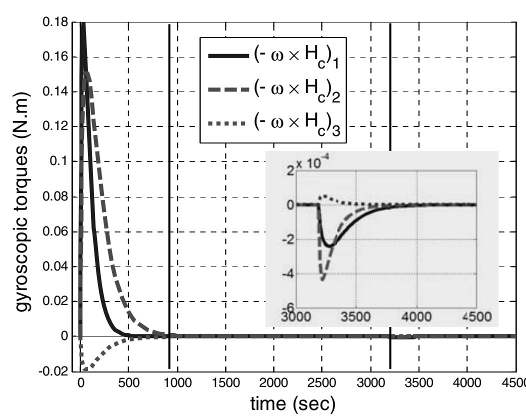
c) Spin rate of MWs



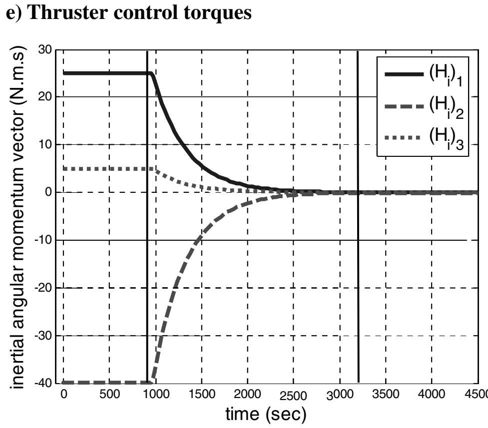
d) Torques of the MW motors



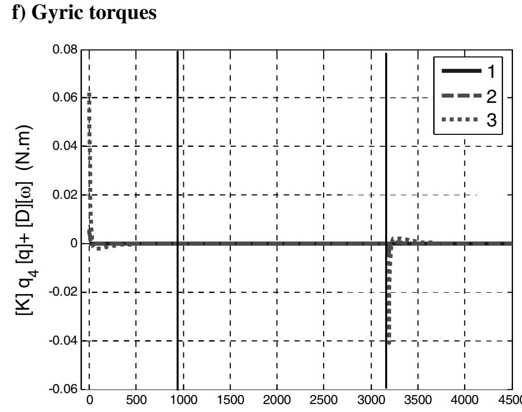
e) Thruster control torques



f) Gyric torques



g) Angular momentum in the inertial frame



h) MW torques, gyric torque subtracted

Fig. 3 Simulation results for momentum dumping using one external control torque.

wheels should cease spinning at the end of the procedure and the spacecraft coincide with the inertial coordinate system again.

The maneuver consists of three phases, separated in Figs. 2a–2h by thick vertical lines. In the first phase, starting at time zero and lasting for about 1300 s, the spacecraft is brought to the intermediate attitude, represented by $\mathbf{q}_{\rm rel} = [-0.2807, -0.3368, 0, 0.8988]^T$, using the momentum wheels. In this orientation, the spinning of the momentum wheel corresponding to the z axis ceases. The second step is angular momentum dumping about x and y axes (the wheel 3 momentum is already dumped), which lasts until the 3300th second. Finally, the spacecraft is brought back to its initial orientation using momentum wheels. Figures 2f and 2h illustrate the contribution of each of the terms in Eq. (16). In the first phase, the gyric torques are significant and high-control torques at MWs are required to compensate them. In contrast, the proportional and derivative torques during the first and the last phases are equal in magnitude, and they have been relatively minute.

B. Momentum Dumping Using One External Control Torque

The angular momentum of the spacecraft can also be removed by external torque actuation about only one principal axis. The initial and desired final conditions are the same as for the simulations in the previous subsection. In this scenario, the sole external torque-actuated axis of the spacecraft is considered as the x axis. Figure 3 illustrates the simulation results for this momentum-dumping maneuver.

The momentum-dumping procedure consists of three steps, separated in Figs. 3a–3h by solid vertical lines. In the first step, lasting about 950 s, the spacecraft is rotated so that its x axis coincides with the direction of the total angular momentum vector. This attitude is represented by $\mathbf{q}_{\rm rel} = [0, -0.0603, -0.4825, 0.8738]^T$. The second and third phases are similar in concept to those described in the previous subsection. Similar to Sec. VI.A, from Figs. 3f and 3h, it can be inferred that the gyric torque is dominant in Eq. (16), and it is way more significant in the first phase than the last.

The simulated momentum-dumping and recovery missions were accomplished in about 75 min. The procedure can be shortened by increasing the gains at the cost of increasing the control torques.

VII. Conclusions

A control method is developed to remove the angular momentum of a spacecraft using either one or two external control torques, when a full set of thruster torques is unavailable due to failure. It is demonstrated that, by performing an attitude maneuver using momentum wheels, followed by implementation of the remaining functional thrusters, it is possible to remove the angular momentum accumulated in the spacecraft. Satisfactory performance of the system is demonstrated by the presented numerical simulations.

Acknowledgments

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